Q) IF A+B+C = π

Then prove that sin2A + sin2B + sin2C = 4sinAsinBsinC

Answer:

LHS

Using formula sin2X=2sinXcosX, we get.

2sinAcosA+2sinBcosB+2sinCcosC....(1)

From given information,

A= π -(B+C),

B= π -(A+C),

C= π -(B+A).

So,

cosA = cos[π -(B+C)] = -cos[B+C] = - [cosBcosC-sinBsinC] ......(2)

cosB = cos[π -(A+C)] = -cos[A+C] = - [cosAcosC-sinAsinC] .....(3)

cosC = cos[π -(B+A)] = -cos[B+A] = - [cosBcosA-sinBsinA] ......(4)

Putting these values in (1) and combining common terms

= -2[sinAcosBcosC+sinBcosCcosA+sinCcosBcosA] + 6sinAsinBsinC.

Putting value for cosBcosC from (2) in first term and taking cosA common from second and 3rd term

= -2[sinA [sinBsinC-cosA] + cosA [sin (B+C)]] + 6sinAsinBsinC.

As A= π -(B+C)

sinA=sin(B+C)

We get after cancellation

= -2 [sinAsinBsinC] + 6sinAsinBsinC.

$= 4\sin A\sin B\sin C$

= RHS

Hence proved